

# Improved Measurement of the Muon Lifetime and Determination of the Fermi Constant

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The MuLan collaboration has measured the lifetime of the positive muon to a precision of 1.0 parts per million. The Fermi constant is determined to a precision of 0.6 parts per million.

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# 1 Introduction

On July 5, 1999 the MuLan collaboration presented the proposal, R-99-07.1: *A precision measurement of the positive muon lifetime using a pulsed muon beam and the MuLan detector*, to the program advisory committee of the Paul Scherrer Institute (PSI). The proposal was to measure the positive muon lifetime to a precision of one part per million (ppm), by which the Fermi constant,  $G_F$ , would be determined to a precision of 0.5 ppm. On December 6, 2010 the collaboration posted at <http://arxiv.org/abs/1010.0991> the manuscript of the paper, which reported on this measurement at the precision of 1.0 ppm. The paper has since been published in Physical Review Letters [1]. The collaboration will prepare and submit for publication a final report of this effort.

# 2 Motivation

The Fermi constant is one of the fundamental constants of the standard model of electroweak interactions. The 1999 proposal was motivated in part by the increased precision of other electroweak parameters, e.g.  $M_Z$ , and in part by the theoretical work of Stuart and von Ritbergen [2], who had undertaken the calculation of two-loop QED radiative corrections, which would reduce the theoretical uncertainty in the extraction of  $G_F$  from the muon lifetime from 15 ppm to approximately 0.2 ppm. In 1999, the dominant error in  $G_F$  was, in fact, from theory and not from experiment. The proposal was also motivated by the opportunity of the intense muon beams of the PSI facility and the opportunity of advanced pulse processing electronics and data acquisition hardware. In a counting experiment a precision of one ppm requires  $10^{12}$  events, a number beyond the capabilities of previous muon lifetime measurements.

# 3 Previous work

At the same PSI advisory committee meeting in July, 1999, the FAST collaboration presented the proposal, R-99-06.1: *Precision measurement of the  $\mu^+$  lifetime ( $G_F$ ) with the FAST detector*, with a similar goal, but with very different methodology. MuLan in 2007 [3] and FAST in 2008 [4] reported 11 ppm and 16 ppm results, respectively, from partial implementations of their respective experiments. Until these measurements, the 18 ppm world average was the result from the experiment of Duclos *et al.* in 1973 (140 ppm), the experiment of Balandin *et al.* in 1974 (36 ppm), the experiment of Bardin *et al.* in 1984 (33 ppm), and the experiment of Giovanetti *et al.* (27 ppm) also in 1984. These experiments separate into the two categories of the “radioactive-source” mode (Duclos *et al.* and Bardin *et al.*) implemented with a pulsed beam, and the “one-at-a-time” mode (Balandin *et al.* and Giovanetti *et*

*al.*) implemented with a low intensity continuous beam. In the “radioactive-source” mode a sample of muons is collected in a stopping medium, and their decay times are measured from an arbitrary start time. In the “one-at-a-time” mode the sample is a single muon. The limiting data rate of the “one-at-a-time” mode is a few tens of kHz, given the  $2.2\ \mu\text{s}$  muon lifetime. The limiting data rate of the “radioactive-source” mode is, in practice, governed by pileup and detector stability considerations. Both the historical and recent muon lifetime measurements are shown in the figure.

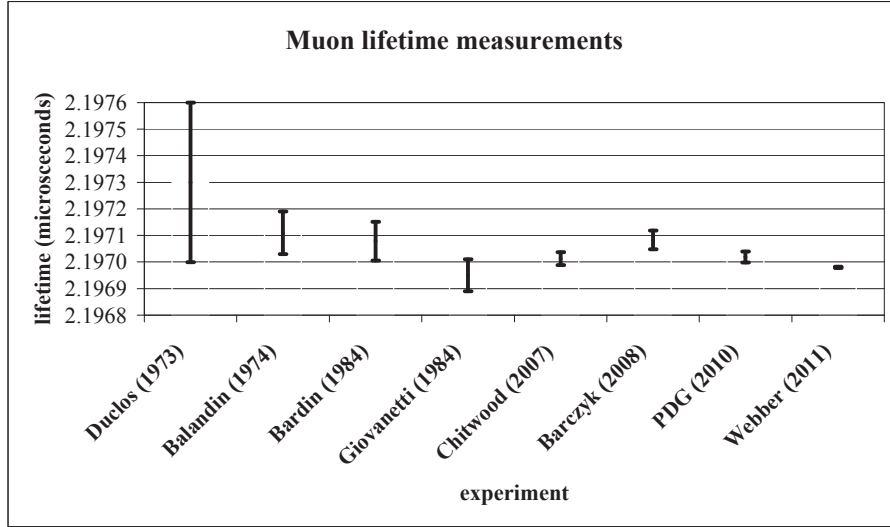


Figure 1: Recent and past muon lifetime measurements.

Statistical considerations aside, the precision of counting experiments is controlled by systematic concerns of counting efficiency stability (affected by both timing stability and gain stability), background level and stability, and pileup. In addition, the muon experiments must mitigate any effect from muon polarization. Muons from pion decay have unit helicity, and muon spin relaxation and/or precession would introduce a time dependence in the decay. In this regard all of the earlier experiments sought to control muon precession with magnetic shielding of the stopping target to a level of tens of  $mG$ . In addition, all of the earlier experiments used detectors symmetrically arranged around the stopping medium and detectors with an effective solid angle with a large fraction of  $4\pi$ .

## 4 The MuLan experiment

Assuming that a running time of approximately two months is acceptable, the required data rate for a one ppm experiment is approximately 200 kHz. Thus the

“radioactive-source” mode must be used at a high intensity facility. PSI has several surface muon beams, *i.e.* muons produced from pions thermalized within the production target, with appropriate intensities. A surface muon beam has a mean momentum of 29 MeV/c and is nearly 100 % polarized. Because of the low momentum, the muons must be transported in vacuum to the stopping target. Given the nominal 50 MHz time structure of the PSI cyclotron, these beams are essentially CW. The MuLan experiment was mounted at the end of the  $\pi$ E3 beam line, a beam line with no permanent end station. There is no pion contamination in this beam line at a mean momentum of 29 MeV/c due to its length of approximately 30 m. There is considerable positron contamination, but positrons are effectively removed with a velocity-selecting  $\vec{E} \times \vec{B}$  filter. The special feature of the beam line as implemented in the MuLan experiment is a custom, 60-ns switching, 25 kV kicker, designed and constructed for the MuLan experiment by TRIUMF. When energized, the muon flux of approximately 10 MHz at the target is reduced by an extinction factor of approximately  $10^3$ . The kicker is switched by external circuitry that synchronizes the data collection cycles into 5  $\mu$ s kicker-off accumulation periods, followed by 22  $\mu$ s kicker-on measurement periods. The kicker voltage during the measurement period determines the time stability of background from this source. Approximately 50 muons are collected in the accumulation period, and approximately 20 muons remain undecayed at the beginning of the measurement period.

Two high statistics data sets were obtained with two different stopping targets, both chosen to mitigate the effects of muon precession and relaxation. The stopping target in the 2006 running period was a 0.5-mm-thick foil of a ferromagnetic alloy, Arnokrome<sup>TM</sup> III (AK-3). (By 2006 the kicker was specified, designed and built, the detector and electronics were built, the targets were chosen and fabricated, and the beam line was understood.) The effective internal field in this material is approximately 0.4 T, which induces a precession with a period of 18 ns. Thus during the accumulation time of 5  $\mu$ s the polarization of the muon ensemble is reduced by a factor of approximately  $10^3$ , as the muons arrive randomly and precess. The stopping target in the 2007 running period was a 2-mm-thick disk of crystalline quartz, in which stopped muons form muonium 90 % of the time. A Halbach arrangement of permanent magnets provides a nearly uniform 130 G field in the plane of the quartz disk. In this field muonium precesses with a period of 2.6 ns. The 10 % muon component in diamagnetic states precesses with a period of 550 ns, which is observed, since the average number of muon rotations during the accumulation period is not large. (Note, however, that the sum of events recorded by a detector at angle  $(\theta, \phi)$  with those from an equally efficient detector at  $(180^\circ - \theta, 180^\circ + \phi)$  form a decay histogram that is immune to precession and relaxation. A detector with point symmetry, viewing a point source, is immune to the effect of muon spin precession and relaxation.)

The detector array consists of 170 stacked pairs of 3-mm-thick plastic scintilla-

tors. They are arranged in a truncated icosahedron geometry and grouped in 20 hexagon and 10 pentagon assemblies. With this segmentation the probability that a pair records an electron during the measurement period is 10 %. Allowing for the beam entry and exit ports and gaps between detector pairs, the detector encompasses 75 % of  $4\pi$  and is point symmetric about the center of the stopping target. The center of a pair of detectors is 38.3 cm from the target. Each scintillator is viewed by either a Photonis or Electron Tubes 29-mm photomultiplier. On average, 80 photoelectrons are registered for each minimum-ionizing particle. The signals from the 340 photomultiplier tubes are recorded using 450 MHz, 8-bit waveform digitizers. The waveform digitizer sampling frequency is set by an Agilent E440 signal generator having stability better than 0.01 ppm/month. The frequency was set to  $\pm 450$  ppm of 451 MHz, with the exact value unknown to the collaboration and only disclosed after the completion of a blind analysis of the two datasets. Thus the photomultiplier waveforms were sampled at a time interval of 2.2 ns (one “clock tick”). Normally 24 ADC samples (53 ns) comprise a full waveform; they are recorded when an input signal exceeds a set threshold with four to eight samples preceding the trigger point. The digitization is extended if the threshold is exceeded at the 24th sample. For the average pulse, the full width at 20 % maximum is 9 ns.

## 5 Data analysis

The 130 TB of raw waveforms are converted into lists of valid “hits”. The waveforms are fit to pulse-shape templates, prepared from a set of low-rate events. The time of a decay is defined as the peak of the pulse shape. In more than 99.9 % of the cases, a single pulse exists on a waveform, and it can be identified reliably. Rare multiple-pulse waveforms are fit by an iterative approach which adds additional pulses as needed. The fitting procedure works reliably when two pulses are separated by more than three “clock ticks.” For shorter separation times, only one hit is reconstructed and its time is set to the pulse-height weighted average of the ADC samples. For resolved pulses, an “artificial deadtime” (ADT) can be applied on a per-detector basis eliminating pulses when a minimum time separates sequential hits in the same scintillator. The ADT used in the analysis varies from 5 to 68 “clock ticks” and represents an important diagnostic of the pileup correction procedure.

Histograms are filled with coincident events from each detector pair; the coincidence window interval is set to the ADT. For the AK-3 data set, the muon lifetime is obtained from a fit to the sum of the 170 individual histograms using the three parameter function

$$F(t) = A \exp(-t/\tau_\mu) + B, \quad (1)$$

where  $B$  accounts for the flat background. A fit to the pileup-corrected event histogram, summed over all detector pairs, gives a  $\chi^2/\text{dof} = 1.03 \pm 0.04$ . For the quartz

data set, in which slow precession and relaxation of the diamagnetic muon component is present, each detector pair histogram is first fit with  $A$  multiplied by the function  $[1 + P_2 \cdot \exp(-t/\tau_2) \sin(\omega t + \phi)]$ ; where  $P_2$  and  $\tau_2$  are transverse polarization and relaxation parameters. The precession and relaxation parameters vary smoothly over the highly uniform detector, and the muon lifetime is obtained from a fit to this distribution. The leading systematic uncertainty in this fitting procedure is from the uncertainty of the beam position on the target. The lifetime obtained in this manner is in agreement to 0.3 ppm with the result from a fit to the simple sum of all detector-pair histograms without regard to precession and relaxation. Despite the complication of muon spin precession and relaxation, a reliable muon lifetime is obtained.

Pileup is corrected using a statistical procedure based on the data itself. Basically, each measurement period presents an identical random sequence of decays. When a hit is observed at a time  $t_i$  in fill  $j$ , an interval between  $t_i$  and  $t_i + \text{ADT}$  is searched in fill  $j + 1$ . If a hit is observed in this interval, its time is recorded in a separate histogram, which is then added back into the original decay histogram. This process is repeated for higher-order pileup and a Monte-Carlo study with the full statistics of the experiment verified the pileup correction procedure. Pileup is found to contribute a systematic error of 0.20 ppm. Gain and timing stability were examined at a similar sensitivity and were found to contribute to the systematic error at a level of 0.25 ppm and 0.12 ppm. The total systematic uncertainty for each running period was 0.42 ppm.

The stability of the lifetime versus the starting time of the fit is a powerful diagnostic because pileup, gain and time stability, and spin precession and relaxation effects might all exhibit time dependencies. For both the 2006 and 2007 data sets the extracted lifetime does not depend on the fit start time, apart from the statistically allowed variation.

## 6 Results

The results for the two running periods are in excellent agreement:

$$\begin{aligned}\tau_\mu(\text{R06}) &= 2196979.9 \pm 2.5 \pm 0.9 \text{ ps}, \\ \tau_\mu(\text{R07}) &= 2196981.2 \pm 3.7 \pm 0.9 \text{ ps}.\end{aligned}\tag{2}$$

As is customary, the first error is statistical and the second is systematic. Combining the two results we obtain

$$\tau_\mu(\text{MuLan}) = 2196980.3 \pm 2.2 \text{ ps} \quad (1.0 \text{ ppm}),\tag{3}$$

The MuLan result is more than 15 times as precise as any other individual measurement and consequently dominates the world average. The MuLan result lies  $2.5 \sigma$

below the current PDG average.

Following the theoretical framework of Stuart and van Ritbergen and the significant emendation of Pak and Czarnecki[5], we obtain the most precise determination of the Fermi constant:

$$G_F(\text{MuLan}) = 1.1663788(7) \times 10^{-5} \text{ GeV}^{-2} \quad (0.6 \text{ ppm}). \quad (4)$$

The Particle Data Group has not (yet) adopted the abovementioned methodology so  $G_F(\text{MuLan})$  should not be directly compared to  $G_F(\text{PDG2010})$ , which was obtained with a different methodology.

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